OPTICAL FORMULAS AND THEIR APPLICATIONS

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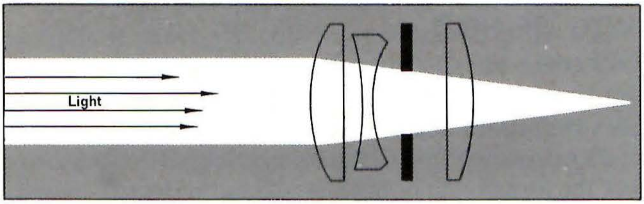


This pamphlet answers some questions about photographic lenses. It explains such terms as *lens-opening markings, depth of field,* and *hyperfocal distance*. It contains a number of optical formulas which you will find useful when you want to know more about some of the factors that control the size and sharpness of the image formed by your camera lens.

LENS DIAPHRAGMS AND THEIR MARKINGS—*f*-NUMBERS

Simple lenses in inexpensive cameras have a fixed lens opening or a series of apertures in a movable slide or disk. More complex lenses have an adjustable lens opening, called a lens diaphragm, that varies the amount of light that passes through the lens. The size of this opening is indicated by a lens-opening (lens-aperture) scale marked in *f*-numbers, also called *f*-stops. Each *f*-number is equal to the focal length of the lens divided by the effective diameter of the lens opening.

The *f*-numbers $f/1$, $f/1.4$, $f/2$, $f/2.8$, $f/4$, $f/5.6$, $f/8$, $f/11$, $f/16$, $f/22$, $f/32$, and $f/45$ are full-stop increments. Each lens opening in the series transmits one-half as much light as the preceding lens opening; for example, $f/5.6$ transmits half as much light as $f/4$. Most lenses do not have a range of openings this great. Sometimes the largest opening for a lens is less than 1 full *f*-stop from the next marked lens opening. Examples are $f/3.5$, $f/4.5$, and $f/6.3$. Most lens diaphragms are continuously variable so that you can set the lens opening at intermediate *f*-numbers for small changes in exposure.

Today most lenses are coated during manufacture to increase light transmittance and reduce lens flare. Coated lenses have a transmittance of about 95 percent, so no exposure correction is required with these lenses.

Effective *f*-Number for Lens Extension. When you use a bellows extension or extension tubes to make extreme close-ups, the lens-to-film distance becomes significantly greater than the focal length. The effective *f*-number is then higher than indicated on the lens-opening scale. To correct for this effect, an exposure compensation is necessary when the subject distance is less than 8 times the focal length of the camera lens. This is especially important when you take pictures on color-slide film or when you use high-contrast black-and-white film for copying work. Exposure in these instances must be close to optimum for the best

pictures. Use the formula for computing effective *f*-number given under Useful Optical Formulas. You can obtain the effective *f*-number more quickly without calculation by using the Lens-Extension Exposure Dial in the *KODAK Master Photoguide* (AR-21), sold by photo dealers.

USING DEPTH OF FIELD

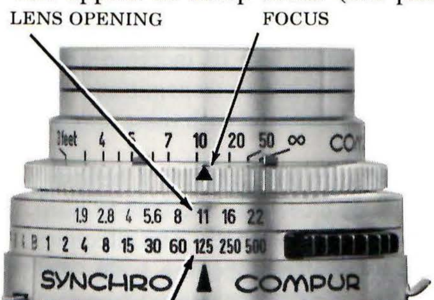
Depth of field is the range of acceptably sharp focus in front of and behind the distance the camera is focused on. By selectively controlling depth of field, you can usually place the emphasis where you want it in your pictures. For example, you can use a larger lens opening to intentionally throw the foreground or background out of focus. To determine the range of sharp focus, consult the depth-of-field scale on your camera, the depth-of-field tables in your camera instruction manual, or the depth-of-field formulas under Useful Optical Formulas.

To get the maximum depth of field for a particular lens opening, focus your camera lens on the hyperfocal distance (defined below). The easiest way to do this with a camera that has a depth-of-field scale is to set the far-limit indicator for the lens opening you are using opposite the infinity mark on the focusing scale. Infinity is usually indicated on the focusing scale by INF or ∞. The infinity setting focuses the lens for distances beyond the maximum distance in feet (or meters) marked on the focusing scale.

HYPERFOCAL DISTANCE

Hyperfocal distance is just a special application of depth of field. When you focus a lens on infinity, the distance beyond which all objects are in acceptably sharp focus is the hyperfocal distance. For example, set a lens having a focal length of 35 mm at *f*/11; then focus the lens on infinity. The depth-of-field scale shows that all objects from 11 feet to infinity will look sharp. The hyperfocal distance is therefore 11 feet.

If you refocus the lens on this hyperfocal distance (11 feet), objects from half of the hyperfocal distance, approximately 6 feet, to infinity will appear in sharp focus (see picture). This gives you the greatest depth of field for that particular lens opening. As you open the lens diaphragm to larger openings, the hyperfocal distance increases and the depth of field decreases.



Many photographers waste depth of field without realizing it. In the example just described, if you focused your camera lens on 50 feet instead of the hyperfocal distance for a subject 50 feet away, your depth of field would be from 10 feet to infinity. This would result in a 4-foot loss in foreground sharpness.

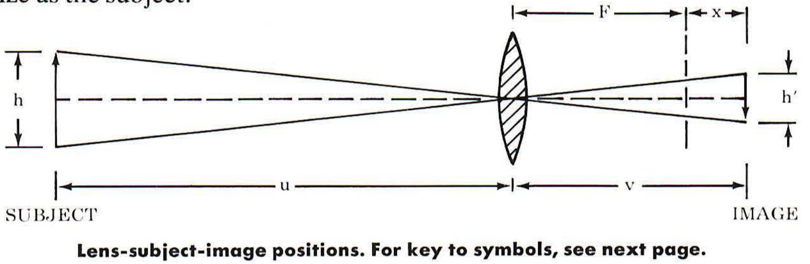
Depth-of-field scales and tables and hyperfocal distances are somewhat arbitrary since they are based on an acceptable circle of confusion. (See Depth of Field under Useful Optical Formulas.) Any point in a scene that is closer or farther away than the distance focused upon will register on the film as a small blurred circle rather than as a point. This blurred circle is called the circle of confusion. When these circles are small enough to appear as points, the subject looks sharp. But the farther a point in the scene is from the plane the camera is focused upon, the larger the circle of confusion becomes. When the circles become so large that the subject no longer appears sharp, it is no longer within the depth of field.

Useful Optical Formulas

In your photographic work, have you at times wanted to know what focal-length lens to use when you have a limited subject distance? How much depth of field you'll have for a certain picture? What close-up lens to use for a particular subject size and image size? How about projection distances for slide and movie projectors? You can answer these and many other questions by using the basic optical formulas on the following pages.

To Find the Focal Length of a Lens. Most lenses have the focal length printed on the front or side of the lens mount. The focal length of a normal camera lens (not a telephoto or wide-angle lens) is equal to the distance from approximately the center of the lens to the image plane when the lens is focused on infinity. If you don't know the focal length of a normal lens, you can measure the approximate focal length from a point midway between the front element and the rear element of the lens. This distance is sufficient for use in the formulas in this pamphlet.

You can determine the focal length of a lens more accurately by first focusing on an object at infinity and then focusing on an object close to the camera to obtain a life-size image (unit magnification). Measure the lens-to-film distance at each setting. The difference between the two lens-to-film distances is equal to the focal length. When you use this technique to find the focal length, your camera must have a bellows or extension tubes that permit extending the lens to obtain an image the same size as the subject.



Lens-subject-image positions. For key to symbols, see next page.

APPROXIMATE POSITION OF SUBJECT AND IMAGE

m = magnification u = subject distance h = height of subject*
F = focal length v = image distance h′ = height of image*
x = distance of image from focal point or distance that lens is extended from infinity setting

All dimensions must be expressed in the same unit of measure.	
To convert dimension in	*divide by*
mm to meters	1000
cm to meters	100
inches to meters	39.4
feet to meters	3.28
mm to inches	25.4

Measuring u and v from a point midway between the front element and the rear element of the lens is accurate enough for practical use with a normal lens (not telephoto or wide-angle). The formulas that do not include v are valid for telephoto lenses and wide-angle lenses when u is large enough so that any inaccuracy in measuring u from the center of the lens is insignificant.

The fundamental relationship between focal length, image distance, and subject distance is

$$\frac{1}{F} = \frac{1}{v} + \frac{1}{u}$$

Formulas that are more directly useful and some examples follow:

Magnification:

$$m = \frac{h'}{h} = \frac{v}{u} = \frac{v-F}{F} = \frac{F}{u-F}$$

Lens-to-Image Distance:

$$v = \frac{Fu}{u-F} = mu = (m+1)F$$

Subject-to-Image Distance:

$$u + v = \frac{(m+1)^2}{m}F$$

Lens-to-Subject Distance:

$$u = \frac{Fv}{v-F} = \frac{v}{m} = \left[\frac{1}{m} + 1\right]F$$

Example 1: How long must a room be for you to photograph groups 10 feet wide when you use a lens with a focal length of 8 inches on a 4 x 5-inch camera?

Solution: Allow 4½ inches for image on horizontal axis of negative.

$$\text{Then } m = \frac{h'}{h} = \frac{4.5}{120} = .038$$

$$\text{and } u = \left[\frac{1}{.038} + 1\right]F =$$

$$(26.3 + 1)F = 27.3 \times 8 =$$
$$218 \text{ inches} = 18+ \text{ feet}$$

This answer gives the lens-to-subject distance. You will also need to add at least 7 feet to allow room for the camera, photographer, background separation, etc. The minimum room length is therefore 25 feet. The room width must be at least 15 feet in order to accommodate the group and lights.

Focal Length:

$$F = \frac{u}{\left[\frac{1}{m} + 1\right]} = \frac{v}{m+1}$$

Example 2: For a room 20 x 32 feet and a 2¼ x 2¼-inch camera, what is the longest focal-length lens feasible for photographing a scene 10 feet wide?

Solution: Since you need about 7 feet of room length for working space, the maximum lens-to-subject distance available is 25 feet or 300 inches; u = 300. You should allow at least ⅛ inch of space on either side of the negative. The usable width of the negative is then 2 inches. Since the width of the subject is 10 feet (120 inches), the magnification (m) equals 2 divided by 120, or .017. The formula now reads:

$$F = \frac{300}{1/.017 + 1} = \frac{300}{59 + 1} =$$

$$\frac{300}{60} = 5$$

Answer: 5 inches (127 mm) is the maximum usable focal length.

*You can use width of the subject for h; then h′ becomes width of the image.

Lens Movement from Infinity Position:

$$x = \frac{F^2}{u-F}$$

Field Size (front-element focusing lenses):

$$\text{Field width} = \text{negative width} \times \frac{u}{F}$$

Effective *f*-Number for Lens Extension: The effective *f*-number is greater than the indicated *f*-number because of the increased image distance (lens-to-film distance). When the subject distance u is less than 8 times the focal length of the camera lens, use one of the following formulas to determine the required exposure compensation. The formulas are valid for any subject distance.

$$\textit{Effective f-number} = \frac{v \times f}{F} = f\,(m + 1)$$

$$\begin{array}{l} \textit{Exposure} \\ \textit{time for lens} \\ \textit{extension} \end{array} = \begin{array}{l} \textit{Normally com-} \\ \textit{puted exposure} \\ \textit{time} \end{array} \times \frac{v^2}{F^2}$$

Where v = lens-to-film distance or focal length plus lens extension from infinity focus, f = *f*-number indicated on lens-opening scale; and F = focal length. For close-up pictures with lens extension, use the effective *f*-number obtained from the first formula when determining your exposure, or compensate your exposure time directly by using the second formula.

DEPTH OF FIELD

Depth-of-field computations are made on the basis of a fixed circle of confusion or on a circle of confusion equal to a fraction of the focal length. Lenses of different focal lengths used at the same *f*-number have the same depth of field for equal image sizes. As a general rule, one-third of the depth of field is in front of the subject and two-thirds is behind the subject. An exception to this rule is extreme close-ups, including those made with close-up lenses, where depth of field is about equal on both sides of the subject.

Method A, Fixed Circle of Confusion:

F = focal length of lens
f = *f*-number setting
H = hyperfocal distance
u = distance for which camera is focused
d = diameter of circle of confusion

Camera	*Fixed Circle of Confusion Most Widely Used (in inches)*
8 mm movie	.0005
Super 8 movie	.00065
16 mm movie	.001
110 (13 x 17 mm*)	.0012
126 (28 x 28 mm*)	.002
135 (24 x 36 mm*)	.002
Roll-film	.005
4 x 5-inch and larger	F/1720 critical use or F/1000 liberal use

$$\textit{Near limit of depth of field}^{\dagger} = \frac{H \times u}{H + (u - F)}$$

$$\textit{Far limit of depth of field}^{\dagger} = \frac{H \times u}{H - (u - F)}$$

*Negative size.
†Measured from camera lens.

Hyperfocal Distance (near limit of depth of field when lens is set at infinity):

$$H = \frac{F^2}{f \times d}$$

Method B, Circle of Confusion a Fraction of the Focal Length of the Lens:

u = distance focused upon
θ = angular size of circle of confusion. For critical definition, θ is 2 minutes of arc and the linear size of the circle of confusion is approximately F/1720 (tan 2′ = .00058).

$$L = \textit{effective diameter of lens} = \frac{F}{f}$$

$$\textit{Near limit of depth of field}^{\ddagger} = \frac{u^2 \tan\theta}{L + u \tan\theta}$$

$$\textit{Far limit of depth of field}^{\ddagger} = \frac{u^2 \tan\theta}{L - u \tan\theta}$$

‡Measured from plane focused upon.

CLOSE-UP LENSES

These formulas will help you to find subject distance, depth of field, and field size when you are using close-up lenses.

The following quantities, except s, must be expressed in meters. The answer will be in meters.

F_s = focal length of close-up lens $= \frac{1}{D}$

D = power in diopters (1+, 2+, 3+, etc) of close-up lens

u = distance from close-up lens to subject

s = focusing-scale setting *in feet*

F_c = combined focal length of camera lens and close-up lens

F = focal length of camera lens

f_c = combined f-number of camera lens and close-up lens—Use for depth-of-field computations only.

f = f-number indicated on lens-opening scale

W = field width

w = negative width

Subject Distance

For Infinity Setting:

$$u = F_s = \frac{1}{D}$$

For two close-up lenses,

$$u = F_s = \frac{1}{D_1 + D_2}$$

For Focusing Scale Set at s Feet:

$$u = \frac{1}{\left. D + \frac{3.28}{s} \right\}*}$$

To find s and D for given u:

$$\frac{1}{u} - D = \frac{3.28}{s}$$

Take highest whole number of D (1, 2, 3, etc) that is not larger than $\frac{1}{u}$. Solve for s.

*3.28/s is power of focusing scale. This is equivalent to the power of a close-up lens which would cause the same change of focus. For example, changing the focusing-scale setting from infinity to 3 feet is equivalent to adding slightly more than one diopter to the power of the close-up lens used.

Combined Focal Length:

$$F_c = \frac{F}{1 + FD}$$

Depth of Field for Given u

Use the hyperfocal distance and depth-of-field formulas on page 5 and substitute F_c for F and f_c for f. Find f_c from the following formula:

$$f_c = \frac{F_c \times f}{F}$$

Use the distance from the close-up lens to the subject for u.

For greatest convenience consult the Depth-of-Field Computer in the *KODAK Master Photoguide.*

Field Size

For Infinity Setting:

$$W = \frac{w\,F_s}{F}$$

For Front-Element Focusing† Lens at s Feet:

$$W = \frac{w}{F\left[D + \frac{3.28}{s}\right]}$$

For Unit-Focusing† Lens at s Feet:

$$W = \frac{\left[\frac{1}{F} - \frac{3.28}{s}\right]}{\left[D + \frac{3.28}{s}\right]}\,w$$

Field height is proportional to negative height.

Wide-Angle Use

Use the following formula for obtaining a wide-angle effect with close-up lenses on view cameras. This technique requires a camera which allows you to use lens-to-film distances shorter than the focal length of the camera lens.

Width of field with close-up lens =

Width of field without close-up lens $\times (1 + FD)$

†Open the back of your camera and look at the lens while adjusting the focus. If the rear lens element moves, your lens is unit focusing; if it doesn't move, your lens is front-element focusing.

More Information

If you have additional questions about optical formulas and their applications, write to Eastman Kodak Company, Photo Information, Department 841, 343 State Street, Rochester, New York 14650.

For more information on lenses, see "A Practical Look at Movie Lenses" in *The Third and Fourth Here's How* (AE-104), $3.50, and "Some Chit-Chat About Lenses" in *More Here's How* (AE-83), $1.50. These books are available from your photo dealer. If not in stock, order by title and code number directly from Eastman Kodak Company, Department 454, Rochester, New York 14650. Please send remittance with your order, including state and local sales taxes. Prices shown are suggested prices only and are subject to change without notice. Actual selling prices are determined by the dealer.

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